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Chemical collision model and statistical distribution function for an anyon

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Letter

CHEMICAL COLLISION MODEL AND STATISTICAL DISTRIBUTION FUNCTION FOR AN ANYON LIQUID

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Contact is established between an approximate chemical collision model and a **full** statistical mechanical treatment of the distribution function of an anyon liquid. The range of validity of the chemical model is thereby delineated.

Keywords: Collision model; anyon distribution function

Early studies by March *et al.* [l] of the statistical distribution function of an anyon liquid, ie., with particles obeying statistics intermediate between Fermions and Bosom, were based on a chemical collision model, motivated by the treatment in Ma's book *[a].* This type of approximate modelling has subsequently been brought to full fruition by a number of authors and we note here especially the study of Wu **[3].** In this work, Wu writes the statistical distribution function *n* for the anyon liquid as

$$
n = \frac{1}{w(\zeta) + \alpha} \tag{1}
$$

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where α is the characteristic anyon parameter, with $\alpha = 0$ corresponding to Bosons and $\alpha = 1$ to Fermions. In eqn (1), ζ is defined by

$$
\zeta = \exp(\beta \{\varepsilon - \mu\}); \quad \beta = 1/k_B T \tag{2}
$$

We will return to Wu's form of the function $w(\zeta)$ below, but our objective here is to compare and contrast the form (1) with the earlier approximate result for *n* based on the chemical collision model already referred to. The shape of the anyon distribution function *n* dictated by the above model is then *[1,4]*

$$
\frac{1}{n} = \exp(\beta \{\varepsilon - \mu\}) + a,\tag{3}
$$

where ϵ is the particle energy. As pointed out in refs 1 and 4, the chemical collision arguments do not contain within themselves a unique procedure for determining the factor *a* in eqn **(3),** which must, of course, be chosen such that $a = -1$ for $\alpha = 0$ and $a = 1$ for $\alpha = 1$, to recover the well-known Bose and Fermi distributions respectively.

By comparing eqns (3) and (l), the anyon factor a in eqn **(3)** can evidently be expressed as

$$
a = w(\zeta) + \alpha - \zeta \tag{4}
$$

where the definition (2) has also been employed. This is the point then at which to introduce the explicit form of $w(\zeta)$ obtained by Wu [3] and other workers *[S, 61,* namely

$$
w^{\alpha}[1+w]^{1-\alpha} = \zeta, \qquad (5)
$$

it being noteworthy in this context that the work of Bhaduri *eta1 [6]* employs the Thomas-Fermi simplification of density functional theory. Using eqn *(3,* one can write the anyon factor a in eqns (3) and **(4)** as

$$
a(\alpha, w) = w + \alpha - w^{\alpha} \left[1 + w \right]^{1 - \alpha}
$$
 (6)

where, as Wu notes, *w* is non-negative.

We turn briefly to examine the limiting case of eqn *(6),* where *w* is large, with α non-zero, the physical range of the anyon parameter

being $0 \le \alpha \le 1$. In this case, with α fixed, for $w \gg \alpha$ one has

$$
a(\alpha, w)|_{w \to \infty} = w + \alpha - w^{\alpha} w^{1-\alpha} \left[1 + \frac{1}{w} \right]^{1-\alpha}
$$

$$
\doteq w + \alpha - w \left[1 + \frac{(1-\alpha)}{w} \right] + \cdots
$$

$$
= 2\alpha - 1 + \cdots \tag{7}
$$

Thus, in this limit, $a(\alpha, w) \rightarrow 2\alpha - 1$, independently of *w* for $w \gg \alpha$. This finding that a is then $a(x)$ was the assumption on which the earlier chemical collision models were utilized [1,4]. Eqn *(7)* already correctly interpolates between the Boson $(\alpha = 0; a = -1)$ and Fermion $(a = 1, a = 1)$ limits. Of course, as *w* becomes small, dependence of *a* on *w* is significant, though $2\alpha - 1$ is not a bad approximation even at $w = 1$ (giving for $\alpha = 1/2$, $a = 0$ instead of 0.09 and for $\alpha = 1/4$, $a = -1/2$ rather than -0.44).

In summary, the collision model of the statistical distribution function $n(\varepsilon, T)$ of an anyon liquid has been brought into intimate contact with a full statistical mechanical treatment $[3,5,6]$. The anyon factor a in the model exemplified by eqn **(3)** has then the rigorous form (6). But for $w \gtrsim 1$, $a \rightarrow 2\alpha - 1$ to a useful approximation; that is a becomes solely dependent on *a,* and not on *w,* which is the simplifiying assumption on which the early chemical collision treatments $[1, 4]$ were constructed.

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