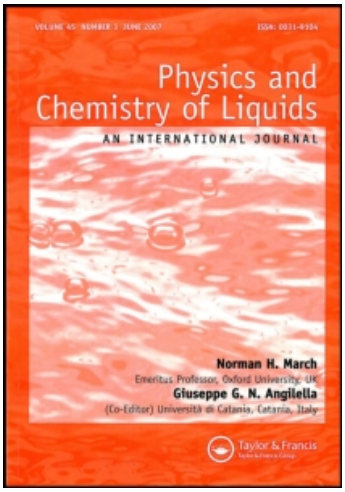

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Chemical collision model and statistical distribution function for an anyon liquid

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Letter

CHEMICAL COLLISION MODEL AND STATISTICAL DISTRIBUTION FUNCTION FOR AN ANYON LIQUID

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Contact is established between an approximate chemical collision model and a full statistical mechanical treatment of the distribution function of an anyon liquid. The range of validity of the chemical model is thereby delineated.

Keywords: Collision model; anyon distribution function

Early studies by March *et al.* [1] of the statistical distribution function of an anyon liquid, ie., with particles obeying statistics intermediate between Fermions and Bosons, were based on a chemical collision model, motivated by the treatment in Ma's book [2]. This type of approximate modelling has subsequently been brought to full fruition by a number of authors and we note here especially the study of Wu [3]. In this work, Wu writes the statistical distribution function n for the anyon liquid as

$$n = \frac{1}{w(\zeta) + \alpha} \tag{1}$$

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where α is the characteristic anyon parameter, with $\alpha = 0$ corresponding to Bosons and $\alpha = 1$ to Fermions. In eqn (1), ζ is defined by

$$\zeta = \exp(\beta\{\varepsilon - \mu\}): \quad \beta = 1/k_B T \tag{2}$$

We will return to Wu's form of the function $w(\zeta)$ below, but our objective here is to compare and contrast the form (1) with the earlier approximate result for n based on the chemical collision model already referred to. The shape of the anyon distribution function n dictated by the above model is then [1,4]

$$\frac{1}{n} = \exp(\beta\{\varepsilon - \mu\}) + a, \tag{3}$$

where ε is the particle energy. As pointed out in refs 1 and 4, the chemical collision arguments do not contain within themselves a unique procedure for determining the factor a in eqn (3), which must, of course, be chosen such that $a = -1$ for $\alpha = 0$ and $a = 1$ for $\alpha = 1$, to recover the well-known Bose and Fermi distributions respectively.

By comparing eqns (3) and (1), the anyon factor a in eqn (3) can evidently be expressed as

$$a = w(\zeta) + \alpha - \zeta \tag{4}$$

where the definition (2) has also been employed. This is the point then at which to introduce the explicit form of $w(\zeta)$ obtained by Wu [3] and other workers [5,6], namely

$$w^{\alpha}[1 + w]^{1-\alpha} = \zeta, \tag{5}$$

it being noteworthy in this context that the work of Bhaduri *et al* [6] employs the Thomas-Fermi simplification of density functional theory. Using eqn (5), one can write the anyon factor a in eqns (3) and (4) as

$$a(\alpha, w) = w + \alpha - w^{\alpha}[1 + w]^{1-\alpha} \tag{6}$$

where, as Wu notes, w is non-negative.

We turn briefly to examine the limiting case of eqn (6), where w is large, with α non-zero, the physical range of the anyon parameter

being $0 \leqslant \alpha \leqslant 1$. In this case, with α fixed, for $w \gg \alpha$ one has

$$a(\alpha, w)|_{w\to\infty} = w + \alpha - w^{\alpha} w^{1-\alpha}\left[1 + \frac{1}{w}\right]^{1-\alpha}$$

$$\doteqdot w + \alpha - w\left[1 + \frac{(1-\alpha)}{w}\right] + \cdots$$

$$= 2\alpha - 1 + \cdots \tag{7}$$

Thus, in this limit, $a(\alpha, w) \to 2\alpha - 1$, independently of w for $w \gg \alpha$. This finding that a is then $a(\alpha)$ was the assumption on which the earlier chemical collision models were utilized [1, 4]. Eqn (7) already correctly interpolates between the Boson ($\alpha = 0: a = -1$) and Fermion ($\alpha = 1, a = 1$) limits. Of course, as w becomes small, dependence of a on w is significant, though $2\alpha - 1$ is not a bad approximation even at $w = 1$ (giving for $\alpha = 1/2$, $a = 0$ instead of 0.09 and for $\alpha = 1/4$, $a = -1/2$ rather than -0.44).

In summary, the collision model of the statistical distribution function $n(\varepsilon, T)$ of an anyon liquid has been brought into intimate contact with a full statistical mechanical treatment [3, 5, 6]. The anyon factor a in the model exemplified by eqn (3) has then the rigorous form (6). But for $w \gtrsim 1$, $a \to 2\alpha - 1$ to a useful approximation; that is a becomes solely dependent on α, and not on w, which is the simplifiying assumption on which the early chemical collision treatments [1, 4] were constructed.

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