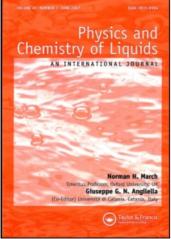
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# Chemical collision model and statistical distribution function for an anyon liquid

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### Letter

## CHEMICAL COLLISION MODEL AND STATISTICAL DISTRIBUTION FUNCTION FOR AN ANYON LIQUID

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Contact is established between an approximate chemical collision model and a full statistical mechanical treatment of the distribution function of an anyon liquid. The range of validity of the chemical model is thereby delineated.

Keywords: Collision model; anyon distribution function

Early studies by March *et al.* [1] of the statistical distribution function of an anyon liquid, ie., with particles obeying statistics intermediate between Fermions and Bosons, were based on a chemical collision model, motivated by the treatment in Ma's book [2]. This type of approximate modelling has subsequently been brought to full fruition by a number of authors and we note here especially the study of Wu [3]. In this work, Wu writes the statistical distribution function n for the anyon liquid as

$$n = \frac{1}{w(\zeta) + \alpha} \tag{1}$$

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where  $\alpha$  is the characteristic anyon parameter, with  $\alpha = 0$  corresponding to Bosons and  $\alpha = 1$  to Fermions. In eqn (1),  $\zeta$  is defined by

$$\zeta = \exp\left(\beta \{\varepsilon - \mu\}\right): \quad \beta = 1/k_B T \tag{2}$$

We will return to Wu's form of the function  $w(\zeta)$  below, but our objective here is to compare and contrast the form (1) with the earlier approximate result for *n* based on the chemical collision model already referred to. The shape of the anyon distribution function *n* dictated by the above model is then [1,4]

$$\frac{1}{n} = \exp\left(\beta\left\{\varepsilon - \mu\right\}\right) + a,\tag{3}$$

where  $\varepsilon$  is the particle energy. As pointed out in refs 1 and 4, the chemical collision arguments do not contain within themselves a unique procedure for determining the factor *a* in eqn (3), which must, of course, be chosen such that a = -1 for  $\alpha = 0$  and a = 1 for  $\alpha = 1$ , to recover the well-known Bose and Fermi distributions respectively.

By comparing eqns (3) and (1), the anyon factor a in eqn (3) can evidently be expressed as

$$a = w(\zeta) + \alpha - \zeta \tag{4}$$

where the definition (2) has also been employed. This is the point then at which to introduce the explicit form of  $w(\zeta)$  obtained by Wu [3] and other workers [5,6], namely

$$w^{\alpha}[1+w]^{1-\alpha} = \zeta,$$
 (5)

it being noteworthy in this context that the work of Bhaduri *et al* [6] employs the Thomas-Fermi simplification of density functional theory. Using eqn (5), one can write the anyon factor a in eqns (3) and (4) as

$$a(\alpha, w) = w + \alpha - w^{\alpha} [1 + w]^{1 - \alpha}$$
(6)

where, as Wu notes, w is non-negative.

We turn briefly to examine the limiting case of eqn (6), where w is large, with  $\alpha$  non-zero, the physical range of the anyon parameter

being  $0 \le \alpha \le 1$ . In this case, with  $\alpha$  fixed, for  $w \gg \alpha$  one has

$$a(\alpha, w)|_{w \to \infty} = w + \alpha - w^{\alpha} w^{1-\alpha} \left[ 1 + \frac{1}{w} \right]^{1-\alpha}$$
  
$$\doteq w + \alpha - w \left[ 1 + \frac{(1-\alpha)}{w} \right] + \cdots$$
  
$$= 2\alpha - 1 + \cdots$$
(7)

Thus, in this limit,  $a(\alpha, w) \rightarrow 2\alpha - 1$ , independently of w for  $w \gg \alpha$ . This finding that a is then  $a(\alpha)$  was the assumption on which the earlier chemical collision models were utilized [1,4]. Eqn (7) already correctly interpolates between the Boson ( $\alpha = 0$ : a = -1) and Fermion ( $\alpha = 1, a = 1$ ) limits. Of course, as w becomes small, dependence of a on w is significant, though  $2\alpha - 1$  is not a bad approximation even at w = 1 (giving for  $\alpha = 1/2$ , a = 0 instead of 0.09 and for  $\alpha = 1/4$ , a = -1/2 rather than -0.44).

In summary, the collision model of the statistical distribution function  $n(\varepsilon, T)$  of an anyon liquid has been brought into intimate contact with a full statistical mechanical treatment [3, 5, 6]. The anyon factor *a* in the model exemplified by eqn (3) has then the rigorous form (6). But for  $w \ge 1$ ,  $a \to 2\alpha - 1$  to a useful approximation; that is *a* becomes solely dependent on  $\alpha$ , and not on *w*, which is the simplifying assumption on which the early chemical collision treatments [1, 4] were constructed.

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